Stochastic Hydroclimatic Modeling and Simulation

Homework-1, Time Series Analysis

1. Assuming that daily measurements of a hydrologic variable *X* can be characterized by the following stationary time series model:

$$X(t) - \mu = \alpha[X(t-1) - \mu] + \epsilon(t) + \beta\epsilon(t-1)$$

where

$$E\big(X(t)\big) = \mu, \quad \epsilon(t) \sim N(0, \sigma_\epsilon^2), \ \forall t \text{ and all } \epsilon's \text{ are independent.}$$

Suppose that measurements of *X* at and prior to *t* are available, and the above model is used to predict *X* at time *t*+*h*, i.e., *x*(*t*+*h*).

(1) What is the 95% prediction interval for $x(t+1)$?

(2) Derive a general expression of the 95% prediction interval for $x(t+h), h \geq 1$.

2. Let the parameters of the model in Problem 1 be as follows:

$$\mu = 17.039, \alpha = 0.9034, \beta = -0.5575, \sigma_\epsilon = 0.3158$$

(1) Generate a realization of $X(t), \ t = 1, 2, \cdots, 220$.

(2) Use simulated series $\{x(1), x(2), \ldots, x(200)\}$ to estimate the parameters of the model.

(3) Calculate the 95% prediction interval for $x(200+h), h = 1, 2, \ldots, 20$.

3. Forty years of daily flow data at the Xia-Yun flow station is available (Xia_Yun_DailyFlow.csv).

(1) Convert the daily flow series to a ten-day period (TDP) data series. [For irrigation water management in Taiwan, TDP flow data are often used.]

(2) Identify and build an AR model for the TDP flow at Xia-Yun station.