

STATISTICS

Homework 8

Due June 18, 2021

1. A factory has thirty production lines for a specific product. All production lines are run under the same process and are independently operated. A new measure was introduced to reduce the product defect ratio from these production lines. The following data represent the defect ratios of these production lines, before and after the measure was implemented.

Production Line	1	2	3	4	5	6	7	8	9	10
Before	8.05	6.00	3.50	5.21	5.06	5.37	5.96	8.49	0.44	3.63
After	6.50	2.21	5.17	4.31	1.34	3.75	4.75	8.14	0.67	1.87
Production Line	11	12	13	14	15	16	17	18	19	20
Before	7.99	4.07	5.55	5.97	4.92	5.35	3.18	6.83	6.39	6.42
After	5.26	3.83	2.98	1.41	5.75	5.02	3.07	3.90	4.67	5.71
Production Line	21	22	23	24	25	26	27	28	29	30
Before	4.93	6.21	5.74	4.17	2.39	1.94	6.95	7.32	6.27	6.30
After	3.90	3.71	4.17	2.88	2.97	1.54	7.44	5.06	2.98	2.84

Conduct a test to see whether implementation of the measure does reduce the defect ratio. Use level of significance $\alpha = 0.05$.

- (1) What is the null hypothesis?
 - (2) What is your test statistic?
 - (3) What is the critical region of the test statistic?
 - (4) What is the p-value of the test statistic based on the above data?
 - (5) What is your conclusion based on the result of your test?
2. The following data represents a random sample from an exponential density with parameter λ $f_X(x; \lambda) = \lambda e^{-\lambda x} I_{(0, \infty)}(x)$, $\lambda > 0$.

9.73	5.12	7.65	4.78	5.81	0.75	5.12	2.90	0.90	5.70
10.25	3.90	3.11	3.85	0.84	3.59	0.34	18.78	5.01	4.73

Conduct a hypothesis test to see whether the null hypothesis $H_0 : \lambda \leq 0.1$ ($H_1 : \lambda > 0.1$) should be rejected at level of significance $\alpha=0.05$.

3. The following is the output of a t-test from R.

```
> x=rnorm( )
> t.test(x,mu=75,alt="less")
```

One Sample t-test

```
data: x
t = , df = 39, p-value = 0.02745
alternative hypothesis: true mean is less than 75
95 percent confidence interval:
 -Inf 74.26614
sample estimates:
mean of x
 70.06415
```

- (1) What is the null hypothesis?

- (2) What is the sample size of the random sample?
 - (3) What is the value of the test statistic calculated from the random sample?
 - (4) What is the sample standard deviation (S_n) of the random sample?
 - (5) Will the null hypothesis be rejected at level of significance $\alpha=0.05$?
4. Data in the following table represents a random sample of final grades of students of statistics class over a few years.

73	61	51	69	66	67	84	87	77	57
81	61	30	23	74	67	64	64	49	35
77	71	70	49	68	52	77	62	62	30
65	62	66	81	71	73	69	27	83	69
72	64	71	67	82					

Suppose that final grades of students in statistics class are normally distributed. At level of significance $\alpha=0.05$, test whether the mean grade is lower than 65 ($H_0 : \mu \geq 65$, $H_1 : \mu < 65$). Develop and draw the power function for the test.

5. Let X and Y be two normal random variables. The following random samples of X and Y are given.

Random sample of X									
80	69	87	63	99	56	21	51	46	43
16	76	80	71	85	77	38	84	53	26
118	63	30	41	57					
Random sample of Y									
87	92	105	68	73	71	97	87	84	67
52	57	92	67	82	85	76	78	71	81

Let μ_X and μ_Y be the expected values of X and Y , respectively. Conduct the following hypothesis test at level of significance $\alpha = 0.05$:

$$H_0: \mu_X - \mu_Y \geq -15$$

$$H_1: \mu_X - \mu_Y < -15$$

6. Suppose that tag numbers (such as TP-000266, TP-018302, TP-010135, TP-020362, ...) of cars registered in a city are issued in series (starting from, for example, TP-000000). A market investigator was asked to estimate the total number of cars registered in the city. The investigator is a person with good knowledge of statistics. He went to the parking lot of a popular shopping mall and recorded the tag numbers of cars (see the following table). If you were the market investigator,
- (1) What is your estimate of the 95% confidence interval of the number of cars registered in the city?
 - (2) Test whether the total number of cars registered in the city is less than 200000.

122693	7594	187456	5060	197346	61076	140399	194731	85065	182
187119	135746	45155	89863	22326	65830	69799	105739	16053	133880
50362	2385	197837	69965	21549	157963	26206	25125	111711	134691
134553	32692	5175	142525	132373	94143	58099	43366	24127	157714
39273	119823	180481	119286	115727	87632	122877	178230	23243	17852
97689	190281	152152	35837	38336	124707	42108	132952	1635	65394
124429	169923	104172	159055	29018	171051	29529	127162	34833	75546
113066	90822	169718	150785	82014	56582	92386	46322	146039	185257
46513	136116	137508	133769	129354	173511	158352	112715	149055	157381
161837	9665	114446	143571	71506	76237	152296	102906	13794	41818