Statistics

Homework 3 (Due April 23, 2021)
Univariate Distributions & Stochastic Simulation

1. Assuming that the number of typhoon occurrences in a year for Taipei follows a Poisson distribution with an expected value of 2.8. The total rainfall amount of a typhoon event has a gamma distribution with the expected value 420mm and standard deviation 360mm. All typhoons are independent. Using R for the following simulation and analyses.
   (1) Simulate 1000 years' typhoon occurrences.
   (2) For each typhoon event, simulate its total rainfall depth.
   (3) Calculate the annual typhoon rainfall amount for each of the 1000 years and plot the frequency histogram of the annual typhoon rainfalls.
   (4) Calculate the mean and standard deviation of the annual typhoon rainfall.
2. Let X be a random variable with a continuous uniform density between -1 and 1, i.e., $X \sim U\,(-1,\quad 1)$. Random variable Y is defined by the following transformation:
   $Y = \frac{2}{\pi}\sqrt{1-X^2}$.
   (1) $Var(Y) = ?$
   (2) $P\left(Y \le \frac{1}{\pi}\right) = ?$
3. Let X be an exponential random variable with the following probability density function:
   $$f_X(x) = \lambda e^{-\lambda x},\ \ \lambda > 0,\ 0 \le x < +\infty.$$
   (1) Find the coefficient of skewness and coefficient of kurtosis of X.
   (2) Generate 1000 random samples, each of size 20, of an exponential random variable with $\lambda = 0.4$. Find the sample coefficient of skewness and sample coefficient of kurtosis of individual random samples using R. [Note: You will need to install the R package "moments" and then use its commands skewness and kurtosis for calculation of sample coefficient of skewness and sample coefficient of kurtosis.]
   (3) Calculate the mean values of the sample coefficient of skewness and sample coefficient of kurtosis.
   (4) Construct the scatter plot of the sample coefficient of skewness and sample coefficient of kurtosis.
4. Prove that the sum of n independently and identically distributed (IID) exponential random variables with parameter λ has a gamma distribution and identify the parameters of the gamma distribution.
5. Let X be a log-normally distributed random variable, i.e. lnX=Y is normally distributed. Prove that the probability density function of X can be expressed as follows:
   $$f_X(x) = \frac{1}{x\sqrt{2\pi}\sigma_Y} e^{-\frac{1}{2}\left(\frac{\ln x-\mu_Y}{\sigma_Y}\right)} \quad [Y \sim N(\mu_Y,\ \sigma_Y^2)]$$

6. The travel time from a student's home to NTU campus involves a few factors:

(i) Walking to the bus stop (stop for traffic lights, crowdedness on the streets, etc.),

(ii) Transportation by buses,

(iii) Walking to NTU campus.

Let X_i be the time required for the i-th factor. X_1 has a normal distribution with a mean of 15 minutes and a standard deviation of 6 minutes. X_2 has a Gamma distribution with a mean of 30 minutes and a standard deviation of 10 minutes. X_3 has a normal distribution with a mean of 10 minutes and a standard deviation of 5 minutes. Assuming that all X_i's are independently distributed.

(1) What is the probability that the student will arrive at NTU campus after 8:30 a.m. if he leaves home at 7:30 a.m.?

(2) When should the student leave home in order to achieve a higher than 90% chance of not being late for a class beginning at 9:10 am?