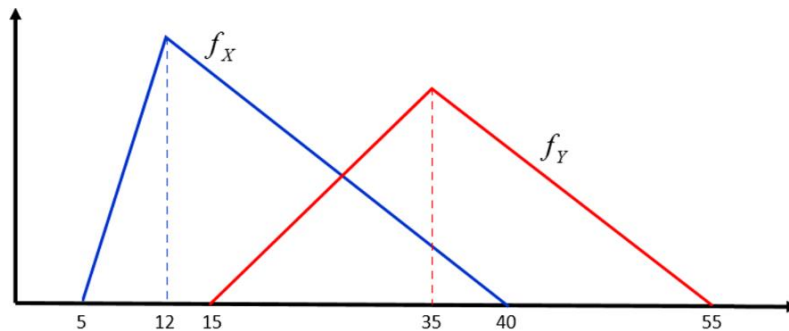


Statistics

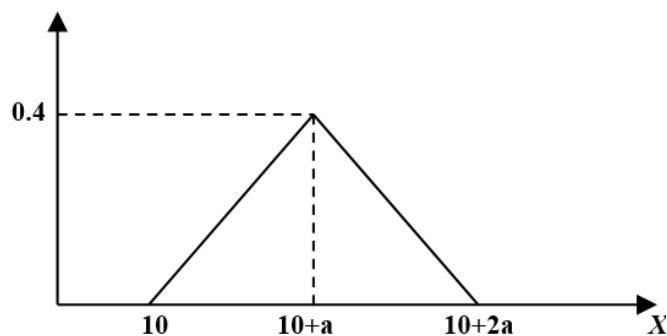
Homework 2 (Due April 2, 2021)

Probability and Random Variable

- An opaque box contains 22 red, 15 yellow, and 13 blue equal-sized cubes. A cube is chosen at random from the box, and it is noted that it is not a blue cube.
 - What is the sample space in view of this knowledge?
 - What is the probability that the selected cube is yellow?
- The function *sample* in R can be used to take a sample of the specified size from the elements of a sample space either with or without replacement.
 - Write an R code using the function *sample* to simulate the random cube drawing of Problem 1.
 - Use the code you developed to draw (with replacement) 10 cubes and show your result.
 - Use the code to draw (without replacement) 10 cubes and show your result.
- A random variable Z is defined as follows.

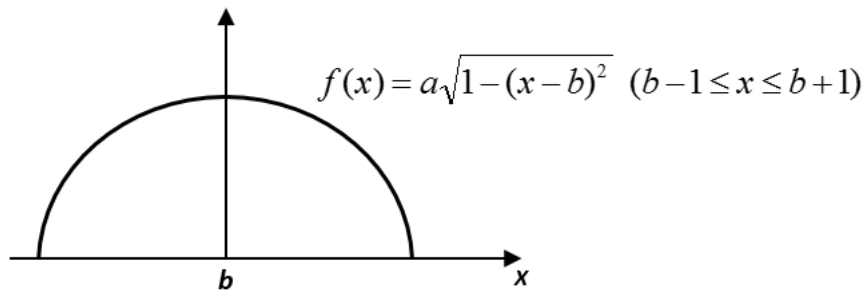


- A random experiment of rolling a die is conducted. $\Omega = \{1, 2, 3, 4, 5, 6\}$
 - If the outcome belongs to $A = \{1, 2\}$, then randomly pick a number from X ;
If the outcome belongs to $B = \{3, 4, 5, 6\}$, then randomly pick a number from Y .
 - Assign the number picked in (2) as an observation of a random variable Z .
- Derive and plot the probability density function of Z .
 - Plot the cumulative distribution function of Z .
- A random variable X has the following probability density function. Random variable Y is the lower-truncated random variable of X with the truncation point at $10+a$.



- Calculate the mean and variance of X .
- What is the probability density function of Y ?
- Calculate the mean and variance of Y .

5. A drunk performs a “random walk” over positions $0, \pm 1, \pm 2, \dots$ as follows. He starts at 0. He takes successive one-unit steps, going to the right with probability p and to the left with probability $1-p$. His steps are independent. Let X denote his position after n steps.
- (1) Find the distribution of $(X+n)/2$.
 - (2) Find the expected value of X .
 - (3) Find the variance of X .
 - (4) If $p=0.6$, what is the probability that after taking $n=20$ steps the drunk will be more than 8 steps away from the beginning location?
 - (5) Develop an R code to simulate the above random walk for $n=20$ steps and plot the paths of 5 simulation runs using the x -axis as the time (steps) and y -axis as the distance from the origin.
6. A random variable X has the following probability density function. Random variable Y is a lower-truncated random variable of X with the truncation point at b .

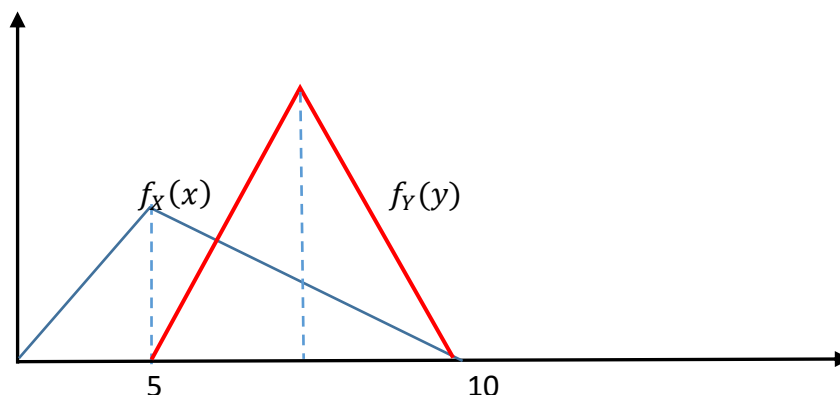


- (1) $a=?$
- (2) What is the probability density function of Y ?
- (3) Calculate the variance of X . [Hint:

$$\int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \left(\frac{x}{a} \right)]$$

- (4) Let $Z=5+3(X-b)$ and calculate the expected value and variance of Z .

7. The probability density functions of two random variables X and Y are shown in the following figure.



- (1) Derive and plot the cumulative distribution functions of X and Y , respectively.
- (2) $P(X > Y)=?$

$$P(X > Y) = \int P(X > y|y)f_Y(y)dy = \int_5^{15} [\int_y^{15} f_X(x)dx]f_Y(y)dy$$