2022 Hydrologic Frequency Analysis
Homework-2

1. The standard error of the method-of-moments (MOM) q-th sample quantile of the Pearson type III (PE3) distribution can be expressed by

$$s_q^2 = \frac{\mu_2}{n}\left\{1 + K_q\gamma + \frac{K_q^2}{2}\left[\frac{3\gamma^2}{4} + 1\right] + 3K_q\frac{\partial K_q}{\partial\gamma}\left[\gamma + \frac{\gamma^3}{4}\right] + 3\left(\frac{\partial K_q}{\partial\gamma}\right)^2\left[2 + 3\gamma^2 + \frac{5\gamma^4}{8}\right]\right\}$$

$$K_q \approx z_q + \left(z_q^2 - 1\right)\frac{\gamma}{6} + \frac{\left(z_q^3 - 6z_q\right)}{3}\left(\frac{\gamma}{6}\right)^2 - \left(z_q^2 - 1\right)\left(\frac{\gamma}{6}\right)^3 + z_q\left(\frac{\gamma}{6}\right)^4 - \frac{1}{3}\left(\frac{\gamma}{6}\right)^5$$

$$\frac{\partial K_q}{\partial\gamma} = \frac{z_q^2 - 1}{6} + \frac{\left(z_q^3 - 6z_q\right)}{54}\gamma - \frac{\left(z_q^2 - 1\right)}{72}\gamma^2 + \frac{z_q}{324}\gamma^3 - \frac{1}{4665.6}\gamma^4$$

   where γ is the coefficient of skewness, z_q is the standard normal deviate ($P(Z \le z_q) = q$).

   Let *X* be a PE3 random variable with the following parameters: Location parameter: 30; scale parameter: 180; shape parameter: 0.64. Also, let X_q be the 100*q*% percentile of *X*.

   (1) Generate 1000 random samples, with sample size *n*=40, from *X*.
   (2) For each of the 1000 random samples, calculate the 70% confidence interval (CI) of X_q for $q = 0.8, 0.9, 0.95, 0.98, 0.99,$ and 0.995.
   (3) Calculate the proportion of the simulated random samples whose 70% CI covers X_q.

2. Conduct the same simulation and analysis for the PE3 random variable described above using the maximum likelihood parameter estimates.
   Note: For maximum likelihood parameters estimation, set the location parameter to be the minimum of the random sample and find the ML estimates of the scale and shape parameters. This is equivalent to maximum likelihood parameter estimation for the gamma distribution.