Geostatistics Fall_2021

Homework 1

1. Given the following AR(1) time series model

   $$X_t = 0.7X_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid\ N(0, \sigma_\varepsilon^2 = 51).$$

   (1) Derive the semi-variogram $\gamma(h)$ for the above AR(1) model.

   (2) In practice, a distance h at which $\gamma(h)$ reaches 95% of its sill is considered as the practical range of a random field. What is the practical range for the above random process?

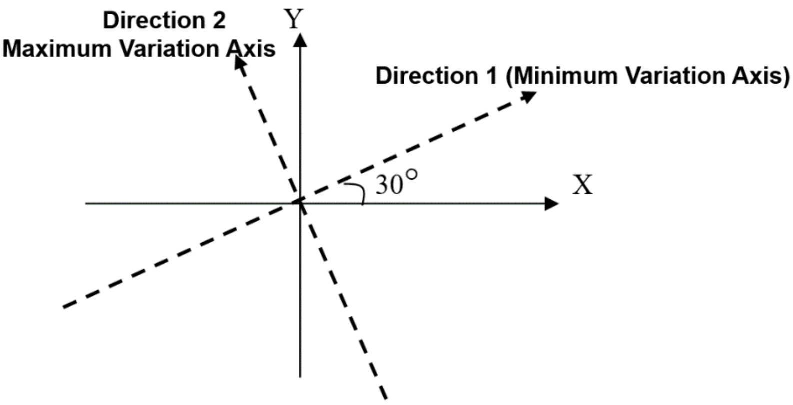
   (3) Let $Y_t = \frac{1}{5}\int_t^{t+5} X(t)dt$. Calculate the variance of Y.

2. Generate 10 sample series (each with a length of 200) of the above time series model, and then derive their corresponding sample series of Y as defined in Problem 1.

   (1) Establish the experimental variogram for each sample series of Y.

   (2) Fit a variogram model to the above experimental variograms.

3. An anisotropic random field $Z(x,y)$ has the following properties:
   - $E[Z(x,y)] = 0$,
   - Anisotropic ratio k (range of the minimum variation axis / range of the maximum variation axis) = 2.0
   - Variogram of Direction 1 (the minimum variation axis) is $\gamma_1(h) = 2.0[1 - \exp(-\frac{h}{4})]$



   (1) $Var[Z(x,y) - Z(x+\sqrt{3}, y+3)]$=?

   (2) $Var[2Z(x,y) - Z(x+4, y+2)]$ =?

4. Let Z(x) be a stationary random field and $E[Z(x)] = 1.0$, $\gamma_Z(h) = 2\left[1 - exp\left(-\frac{h}{2}\right)\right]$.

   Another random field Y(x) is defined as $Y(x) = 3Z(x) - 5Z(x+3)$.

   (1) Var(Y)=?

   (2) Find the semi-variogram of the random field Y, i.e., $\gamma_Y(h)$ =?

5. Use the hourly rainfall data in the Typhoon.csv to conduct the following analyses:

   (1) Calculate and plot the experimental semivariogram cloud of the 18th hourly rainfalls.

   (2) Calculate and plot the experimental semivariogram cloud of the 10th hourly rainfalls.