

## Working Problems for BSE 5034 Stochastic Hydrology (2020)

### WP-5 Time Series Analysis

1. Assuming that daily measurements of a hydrologic variable  $X$  can be characterized by the following stationary time series model:

$$X(t) - \mu = \alpha[X(t-1) - \mu] + \epsilon(t) + \beta\epsilon(t-1)$$

where

$$E(X(t)) = \mu, \quad \epsilon(t) \sim N(0, \sigma_\epsilon^2), \quad \forall t \text{ and all } \epsilon's \text{ are independent.}$$

Supposed that measurements of  $X$  at and prior to  $t$  are available and the above model is used for prediction of  $X$  at time  $t+h$ , i.e.  $x(t+h)$ .

- (1) What is the 95% prediction interval for  $x(t+1)$ ?
  - (2) Derive a general expression of the 95% prediction interval for  $x(t+h)$ ,  $h \geq 1$ .
2. Let the parameters of the model in Problem 1 be as follows:

$$\mu = 17.039, \alpha = 0.9034, \beta = -0.5575, \sigma_\epsilon = 0.3158$$

- (1) Generate a realization of  $X(t)$ ,  $t = 1, 2, \dots, 220$ .
  - (2) Use simulated series  $\{x(1), x(2), \dots, x(200)\}$  to estimate the parameters of the model.
  - (3) Calculate the 95% prediction interval for  $x(200+h)$ ,  $h = 1, 2, \dots, 20$ .
3. Forty years of daily flow data at the Xia-Yun flow station is available (Xia\_Yun\_DailyFlow.csv).
    - (1) Convert the daily flow series to 10-day period (TDP) data series. [For irrigation water management in Taiwan, TDP flow data are often used.]
    - (2) Identify and build an AR model for the TDP flow at Xia-Yun station.