Working Problems for BSE 5034 Stochastic Hydrology (2020)

WP-1 Stochastic simulation of univariate distributions

1. If $U \sim U[0,1]$, show that $-\ln U = \ln\left(\frac{1}{U}\right)$ is exponentially distributed. [Probability integral transformation]

2. Proof and practice of the Acceptance/Rejection method for random number generation of the standard normal distribution.

The exponential density $g(x) = \lambda e^{-\lambda x}$ $(x > 0)$ with $\lambda = 1$ is everywhere higher than the standard normal density $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ for $0 \le x < +\infty$. Prove and show that the acceptance/rejection method with $cg(x) = \sqrt{\frac{2e}{\pi}} e^{-x}$ $(x > 0)$ can be used to generate random samples of the standard normal distribution.

3. Generate a random sample of size 100 from a Pearson Type III distribution of the following parameters: location parameter = 20, scale parameter = 120, shape parameter = 2.

(i) By using the frequency factor approach

(ii) By using rgamma in R.

4. Let X and Y form a bivariate standard normal distribution with a correlation coefficient of ρ.

(i) Derive the algorithm of simulating random samples of a bivariate normal distribution using the concept of principal component analysis.

(ii) Generate a random sample of size 1000 from a bivariate standard normal distribution with $\rho = 0.7$ by using the above algorithm.