Geostatistics Fall_2010

Homework 1

1. Let random variables X and Y be normally and exponentially distributed, respectively.
   (i). What are their corresponding probability density functions?
   (ii). How many parameters are there for probability density functions of X and Y?
   (iii). For each of the two random variables, express the first and second moments in terms of their mean and standard deviation.
2. Consider a stationary random process $X(t) = 12 + 0.64X(t-1) + 0.16X(t-2) + \varepsilon(t)$ where $\varepsilon(t)$'s are independently and identically distributed (*IID*) noise with *PDF* $N(\mu = 0, \sigma_{\varepsilon}^2 = 25)$.
   (i). Calculate the mean and variance of $X(t)$.
   (ii). Calculate $Cov[X(t), X(t-1)]$ and $Cov[X(t), X(t-2)]$.
   (iii). Calculate $Cov[X(t), \varepsilon(t-1)]$.
3. In the literature there are studies that consider the variation of earth surface elevation (sea floor and ground surface) as a random field. However, accurate elevation survey is also an essential and important task for any engineering project. Apparently, the surface elevation at a certain location is not changing and has a fixed elevation (otherwise there is no need for land surface surveying). Make your comment on modeling the earth surface elevation by a random field.
4. Generate a random sample of size 200 of a random variable X with cumulative distribution function $F_X(x) = 1 - e^{-x}$. Calculate the sample mean and standard deviation. Also sketch the empirical cumulative distribution function.
5. Let $X_{\tau}(t)$ represent the position of a particle performing random walk on a real line with step length $\delta = \sqrt{\tau}$ at every time interval τ. Derive the covariance and correlation functions of $X_{\tau}(t)$, i.e.

   $Cov\left[X_{\tau}(t), X_{\tau}(t+h)\right] = ?$

   $Correl\left[X_{\tau}(t), X_{\tau}(t+h)\right] = ?$