Working Problems for BSE 5034 Stochastic Hydrology (2020)

WP-2 Fundamentals of Hydrological Frequency Analysis (HFA-1)

(1) Comparing estimators of coefficient of skewness for the Gamma (Pearson Type III) distribution. (Evaluating the unbiasedness and mean square error)
For a gamma random variable having the following parameters: scale parameter = 70.251, shape parameter = 1.497.

(i) Generate 10,000 sets of random sample, each of sample size n=60.

(ii) Evaluate the bias and mean-square-error of different estimators of coefficient of skewness using the samples generated in (i).

(2) Probability plotting.
A random sample of size 100 is given in the following table.

45.66	55.78	63.85	69.10	69.87	70.63	73.71	74.27	74.54	74.76
74.88	75.76	75.98	78.14	78.22	79.31	80.01	81.51	82.41	84.40
86.02	86.62	86.80	88.10	88.34	91.68	91.86	92.07	92.26	93.52
94.46	95.17	95.72	97.14	97.74	97.87	97.96	99.37	99.76	100.06
101.42	102.00	102.52	102.86	104.12	104.69	104.85	104.97	105.64	105.77
106.79	108.83	108.89	109.67	109.77	110.97	112.68	112.79	114.97	119.52
120.26	121.12	121.31	123.36	126.07	126.26	126.89	128.19	128.29	130.30
133.74	134.72	135.94	139.43	140.05	141.05	141.34	141.63	142.18	144.11
144.18	145.30	146.17	151.33	154.61	158.73	158.74	161.17	161.66	163.19
166.09	174.06	183.49	184.24	190.82	192.48	197.15	215.90	240.14	270.20

Use the probability plotting method to see whether the above data can be characterized by (i) the Gumbel distribution and (ii) the normal distribution.
[Note] The evd package in R (with dgumbel, rgumbel, pgumbel, and qgumbel) can be used for stochastic simulation of the Gumbel distribution.

(3) Let $(x_1,\ x_2,\cdots,x_n)$ be a random sample of size *n* from a distribution with a cumulative distribution function $F_X(\cdot)$ and $x_1 < x_2 < \cdots < x_n$. The empirical cumulative distribution function (ECDF) based on the random sample is defined as

$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{(-\infty,x]}(x_i);\ -\infty < x < \infty$. Prove the following results.

(i) $F_n(X)$ is asymptotically normal, i.e., $F_n(X)$ approaches a normal distribution as *n* approaches infinity.

(ii) $E[F_n(X)] = F_X(X)$

(iii) $Var[F_n(X)] = \frac{1}{n} F_X(X)[1 - F_X(X)]$

(iv) $\sqrt{n}[F_n(X) - F_X(X)]$ approaches a normal distribution with mean 0 and variance $F_X(X)[1 - F_X(X)]$ as *n* approaches infinity.