

Geostatistics Fall_2020

Homework 2

1. Given the following AR(1) time series model

$$X_t = 0.7X_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid N(0, \sigma_\varepsilon^2 = 51).$$

- (1) Derive the semi-variogram $\gamma(h)$ for the above AR(1) model.
 (2) In practice, a distance h at which $\gamma(h)$ reaches 95% of its sill is considered as the practical range of a random field. What is the practical range for the above random process?

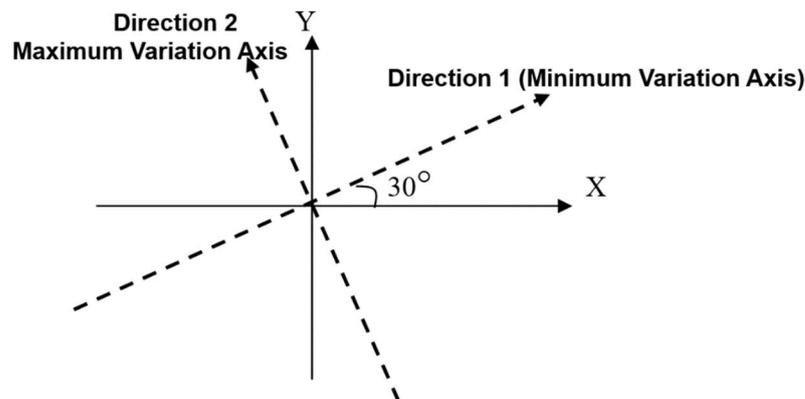
(3) Let $Y_t = \frac{1}{5} \int_t^{t+5} X(t) dt$. Calculate the variance of Y .

2. Generate 10 sample series (each with a length of 200) of the above time series model, and then derive their corresponding sample series of Y as defined in Problem 1.

- (1) Establish the experimental variogram for each sample series of Y .
 (2) Fit a variogram model to the above experimental variograms.

3. An anisotropic random field $Z(x,y)$ has the following properties:

- $E[Z(x, y)] = 0$,
- Anisotropic ratio k (range of the minimum variation axis / range of the maximum variation axis) = 2.0
- Variogram of Direction 1 (the minimum variation axis) is $\gamma_1(h) = 2.0[1 - \exp(-\frac{h}{4})]$



- (1) $Var[Z(x, y) - Z(x + \sqrt{3}, y + 3)] = ?$
 (2) $Var[2Z(x, y) - Z(x + 4, y + 2)] = ?$

4. Let $Z(x)$ be a stationary random field and $E[Z(x)] = 1.0$, $\gamma_Z(h) = 2 \left[1 - \exp\left(-\frac{h}{2}\right) \right]$.

Another random field $Y(x)$ is defined as $Y(x) = 3Z(x) - 5Z(x + 3)$.

- (1) $Var(Y) = ?$
 (2) Find the semi-variogram of the random field Y , i.e., $\gamma_Y(h) = ?$
5. Use the hourly rainfall data in the Typhoon.csv to conduct the following analyses:
- (1) Calculate and plot the experimental semivariogram cloud of the 18th hourly rainfalls.
 (2) Calculate and plot the experimental semivariogram cloud of the 10th hourly rainfalls.