

STATISTICS

Homework 8

Due 6/21/2019

1. A sample was drawn from each of five populations assumed to be normal with the same variance. The values of $(n-1)S^2 = \sum_{i=1}^n (X_i - \bar{X}_n)^2$ and n , the sample size,

were

S^2	40	30	20	42	50
n	6	4	3	7	8

- (1) Find a 95% confidence interval for the common variance σ^2 .
 - (2) At level of significance $\alpha = 0.05$, conduct the following hypothesis test :
 $H_0 : \sigma^2 \geq 50$,
 $H_1 : \sigma^2 < 50$.
2. The data in the following table is a random sample from an exponential density $f_X(x; \lambda) = \lambda e^{-\lambda x}$.

62.48	10.37	31.19	9.57	85.27	162.96	16.11	133.86	47.28	34.79
70.15	75.26	25.90	9.34	47.28	4.33	35.09	48.69	83.99	158.74
60.66	56.81	64.01	89.74	7.02	64.72	100.14	16.64	147.06	28.00
116.40	11.01	29.21	12.54	43.04	18.69	50.47	17.47	56.22	25.18

- (1) Conduct the following hypotheses test at level of significance $\alpha = 0.05$:
 $H_0 : \lambda \leq \frac{1}{45}$, $H_1 : \lambda > \frac{1}{45}$ (considering $n\lambda\bar{X}_n$ as a pivotal quantity)
 - (2) Find the corresponding confidence interval of λ .
3. Use the same data in problem 2.
- (1) Conduct the following hypotheses test at level of significance $\alpha = 0.05$:
 $H_0 : \lambda \leq \frac{1}{45}$, $H_1 : \lambda > \frac{1}{45}$ (considering $n\lambda Y_1$ as a pivotal quantity, where Y_1 is an order statistic.)
 - (2) Find the corresponding confidence interval of λ .
4. A random sample of size 20 $((x_1, x_2, \dots, x_n), n = 20)$ drawn from a continuous uniform density $U[0, \theta]$ is shown in the following table.

11.90	6.94	14.90	4.36	14.00
15.21	18.87	4.75	7.67	6.57
5.15	14.83	7.79	13.88	18.55
2.49	9.90	9.68	11.10	9.92

Conduct the following test at level of significance $\alpha = 0.05$:

$$H_0 : \theta \leq 20, \quad H_1 : \theta > 20 \text{ (considering } Y_n / \theta \text{ as a pivotal quantity).}$$

5. A random sample from a continuous uniform distribution $U[0, \theta]$ is given in the following table. A hypothesis test $H_0 : \theta = 5$, $H_1 : \theta \neq 5$ is to be conducted.

0.9008	3.5321	4.9546	2.9965	4.6778
2.2499	3.4768	0.295	3.8101	3.7739
2.7089	2.7323	0.1799	2.1793	2.5436

1.545	4.8106	1.4242	2.0554	4.5775
4.6189	2.3098	1.8262	1.2262	4.8855

- (1) Conduct the test using $T = \sum_{i=1}^n \left[-\ln\left(\frac{X_i}{5}\right) \right]$ as a test statistic and level of significance $\alpha = 0.05$.
- (2) Conduct the test using $T = \frac{Y_n}{5}$ and level of significance $\alpha = 0.05$.