

STATISTICS

Homework 4 (Due 4/26/2019)

1. Let X_i denote the number of meteors that collide with a test satellite during the i th orbit. Let $S_n = \sum_{i=1}^n X_i$; that is, S_n is the total number of meteors that collide with the satellite during n orbits. Assume that the X_i 's are independent and identically distributed Poisson random variables having mean λ .

(1) Find $E[S_n]$ and $\text{Var}[S_n]$.

(2) If $n=100$ and $\lambda=4$, find approximately $P[S_{100} > 400]$.

2. Random variables X_1, X_2, \dots, X_n have a common density $f_X(\cdot)$ with variance σ^2 .

Let $\bar{X}_n = (X_1 + X_2 + \dots + X_n)/n$. Prove that

$$\text{Var}[\bar{X}_n] = \frac{\sigma^2}{n} + \frac{2}{n^2} \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{Cov}(X_i, X_j).$$

3. Two normal random variables X and Y are jointly distributed with $\text{Var}(X) = 25$ and $\text{Var}(Y) = 1600$. It is known that $P(Y > 80 | X = 50) = 0.1$ and $P(Y > 22 | X = 40) = 0.7886$.

(1) What is the correlation coefficient between X and Y ?

(2) What is the expected value of Y given $X = 50$?

4. Assume that the total rainfall amount of a typhoon event can be characterized by a gamma distribution with expected value 450mm and standard deviation 250mm. The annual count of typhoon events follows a Poisson distribution with expected value 3.2.

(1) Simulated 1000 years of typhoon occurrence and calculate the annual typhoon rainfalls (R) of individual years.

(2) $P(R \geq 1600\text{mm}) = ?$

(3) $E(R) = ?$