

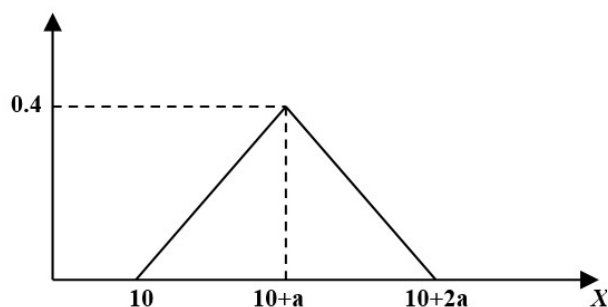
## Statistics - Homework 2 (Due March 22, 2019)

1. Find a constant  $b > 0$  so that the function

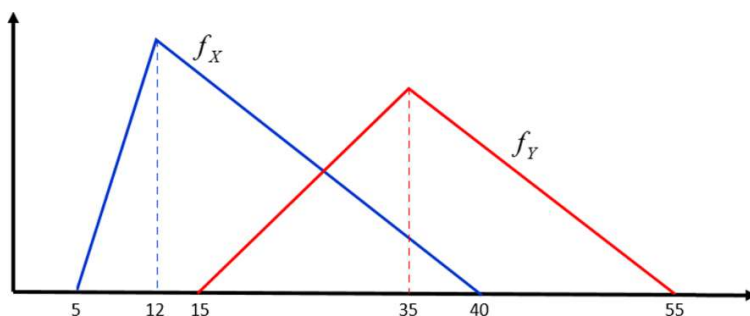
$$f_X(x) = \begin{cases} e^{3x}/4 & 0 \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

is a valid probability density function.

2. A random variable  $X$  has the following probability density function. Random variable  $Y$  is the lower-truncated random variable of  $X$  with the truncation point at  $10+a$ .



- (1) Calculate the mean and variance of  $X$ .
  - (2) What is the probability density function of  $Y$ ?
  - (3) Calculate the mean and variance of  $Y$ .
3. A drunk performs a “random walk” over positions  $0, \pm 1, \pm 2, \dots$  as follow. He starts at 0. He takes successive one-unit steps, going to the right with probability  $p$  and to the left with probability  $1-p$ . His steps are independent. Let  $X$  denote his position after  $n$  steps.
- (1) Find the distribution of  $(X+n)/2$ .
  - (2) Find the expected value of  $X$ .
  - (3) Find the variance of  $X$ .
4. A random variable  $Z$  is defined as follows.



- (1) A random experiment of rolling a die is conducted.  $\Omega = \{1, 2, 3, 4, 5, 6\}$
- (2) If the outcome belongs to  $A = \{1, 2\}$ , then randomly pick a number from  $X$ ;  
If the outcome belongs to  $B = \{3, 4, 5, 6\}$ , then randomly pick a number from  $Y$ .
- (3) Assign the number picked in (2) as an observation of a random variable  $Z$ .

- (1) Derive and plot the probability density function of  $Z$ .
- (2) Plot the cumulative distribution function of  $Z$ .

5. Let  $X$  represent the time delay (in seconds) that a motorist needs to wait after making a required stop at a traffic stop sign. The cumulative distribution function of  $X$  is shown in the following figure.

- (1) Derive and plot the probability density of  $X$ .
- (2)  $P[X < 20] = ?$
- (3)  $P[10 < X < 40] = ?$

Cumulative Probability

