

Working Problems for BSE 5034 Stochastic Hydrology (2017)

WP-1 Stochastic simulation of univariate distributions

- (1) If $U \sim U[0,1]$, show that $-\ln U$ is exponentially distributed. [Probability integral transformation]
- (2) Proof and practice of the Acceptance/Rejection method for random number generation of the standard normal distribution.

The exponential density $f_E(x) = \lambda e^{-\lambda x}$ with $\lambda = 1$ is everywhere higher than the standard normal

$$\text{density } f_N(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \text{ for } 0 \leq x < +\infty.$$

Prove and show that the acceptance/rejection method

with $g(x) = \sqrt{\frac{2e}{\pi}} e^{-x}$ can be used to generate random

samples of the normal distribution.

- (3) Generate a random sample of size 100 from a Pearson Type III distribution of the following parameters: location parameter = 20, scale parameter = 120, shape parameter = 2.
 - (i) By using the frequency factor approach
 - (ii) By using rgamma in R.