Statistics - Homework 8 (Due Jan. 10, 2018)

1. An Analyst draws a random sample of size 8, $X_1,\cdots,X_8$, from a distribution with the probability density function $f(x;\theta)=(\theta+1)x^{\theta}$, $0\le x\le 1$, $\theta>0$. The analyst wants to test $H_0:\theta=2$ versus $H_0:\theta>2$. As a decision rule, she records Y, the number of observations which are greater than or equal to 0.88. She rejects the null hypothesis if $Y>5$. What is the probability of a Type I error?

2. A political candidate wants to conduct a survey to gauge voters' support. He would like a confidence interval with an error of no more than 4%, with 95% confidence.
   (1) How large should the sample size be for the survey?
   (2) Using the sample size determined in (1), calculate the lengths of the 95% confidence interval, if the candidate's support rates ($\hat{p}$) are estimated as 5%, 10%,…, 95%, respectively.
   (3) Plot the margin of errors (half of the length of the confidence interval) with respect to the estimated support rates.
   (4) Based on the survey results, if the candidate's (candidate A) support rate is 27% and another candidate's (candidate B) support rate is 20%, are you convinced that candidate A leads in the voter's support, with 95% confidence?

3. A random sample of size 35 is shown in the following table.

29.32	32.84	8.98	40.61	17.70
33.07	60.90	9.29	11.85	25.28
44.57	15.08	4.31	41.93	58.78
21.64	35.57	19.36	19.86	44.51
24.35	34.09	19.24	28.39	41.93
8.66	29.01	36.48	25.78	18.74
11.33	15.83	26.17	45.43	6.64

Use the chi-square goodness-of-fit test (at level of significance α= 0.05) to test whether the above data is originated from a gamma density. [Note: You need to estimate the parameters of the gamma density using the method of moments.]

4. A total of 30 soil samples were randomly and independently taken from a study site for measurement of heavy metal Cd (Cadmium) concentration. Measurements of Cd concentration (mg/kg) is listed in the following table:

0.66	0.39	0.76	0.19	0.03	0.5	0.19	0.19	0.15	0.85
0.82	0.46	0.58	0.07	0.07	0.23	0.07	0.07	0.5	0.77
0.35	0.62	0.46	0.46	0.54	0.15	0.85	0.23	0.46	0.38

Cleanup standard for Cd concentration is 1 mg/kg, i.e., the study site is considered not contaminated if and only if the population mean in the study site is lower than 1 mg/kg. Conduct the test $H_0:\mu\ge 1$ vs $H_1:\mu<1$. (Assume variance of the Cd concentration σ^2 = 0.0676 $(\text{mg/kg})^2$ and let the size of the test be α=0.05.)

5. The data in the following table is a random sample from an exponential density $f_X(x;\ \lambda) = \lambda e^{-\lambda x}$.

62.48	10.37	31.19	9.57	85.27	162.96	16.11	133.86	47.28	34.79
70.15	75.26	25.90	9.34	47.28	4.33	35.09	48.69	83.99	158.74
60.66	56.81	64.01	89.74	7.02	64.72	100.14	16.64	147.06	28.00
116.40	11.01	29.21	12.54	43.04	18.69	50.47	17.47	56.22	25.18

(1) Conduct the following hypotheses test at level of significance $\alpha = 0.05$:

$H_0 : \lambda \le \frac{1}{45}, \quad H_1 : \lambda > \frac{1}{45}$ (considering $n\lambda\overline{X}_n$ as a pivotal quantity)

(2) Find the corresponding confidence interval of λ.

(3) Calculate and plot the power function of the test.

6. Use the same data in problem 7.

(1) Conduct the following hypotheses test at level of significance $\alpha = 0.05$:

$H_0 : \lambda \le \frac{1}{45}, \quad H_1 : \lambda > \frac{1}{45}$ (considering $n\lambda Y_1$ as a pivotal quantity, where Y_1 is an order statistic.)

(2) Find the corresponding confidence interval of λ.

(3) Calculate and plot the power function of the test.

7. A random sample of size 20 ($(x_1, x_2, \cdots, x_n), \quad n = 20$) drawn from a uniform density $U[0, \theta]$ is shown in the following table.

11.90	6.94	14.90	4.36	14.00
15.21	18.87	4.75	7.67	6.57
5.15	14.83	7.79	13.88	18.55
2.49	9.90	9.68	11.10	9.92

(1) Conduct the following test at level of significance $\alpha = 0.05$:

$H_0 : \theta \le 20, \quad H_1 : \theta > 20$ (considering Y_n / θ as a pivotal quantity).

(2) Calculate and plot the power function of the test.