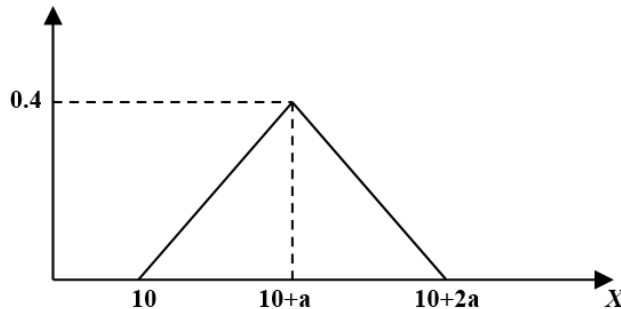


Statistics - Homework 2 (Due Oct. 18, 2017)

1. Calculate the following probabilities:

- (1) Out of a group of 25 persons, what is the probability that at least two persons in the group will have the same birthday? (Assume a 365-day year and that all days are equal likely.)
- (2) Out of a group of 50 persons, what is the probability that at least two persons in the group will have the same birthday?

2. A random variable X has the following probability density function. Random variable Y is the lower-truncated random variable of X with the truncation point at $10+a$.

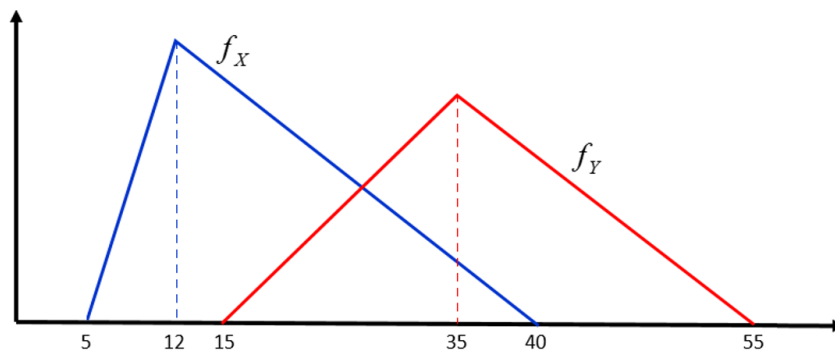


- (1) Calculate the mean and variance of X .
- (2) What is the probability density function of Y ?
- (3) Calculate the mean and variance of Y .

3. A drunk performs a “random walk” over positions $0, \pm 1, \pm 2, \dots$ as follow. He starts at 0. He takes successive one-unit steps, going to the right with probability p and to the left with probability $1-p$. His steps are independent. Let X denote his position after n steps.

- (1) Find the distribution of $(X+n)/2$.
- (2) Find the expected value of X .
- (3) Find the variance of X .

4. A random variable Z is defined as follows.



- (1) A random experiment of rolling a die is conducted. $\Omega = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}\}$
- (2) If the outcome belongs to $A = \{\mathbf{1}, \mathbf{2}\}$, then randomly pick a number from X ; If the outcome belongs to $B = \{\mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}\}$, then randomly pick a number from Y .
- (3) Assign the number picked in (2) as an observation of a random variable Z .

- (1) Derive and plot the probability density function of Z .
- (2) Plot the probability cumulative distribution of Z .

5. Let X represent the time delay (in seconds) that a motorist needs to wait after making a required stop at a traffic stop sign. The cumulative distribution function of X is shown in the following figure.

(1) Derive and plot the probability density of X .

(2) $P[X < 20] = ?$

(3) $P[10 < X < 40] = ?$

Cumulative Probability

