STATISTICS

Homework 9

Due December 31, 2015

1. A sample was drawn from each of five populations assumed to be normal with the same variance. The values of $(n-1)S^2 = \sum_{i=1}^{n}(X_i - \overline{X}_n)^2$ and *n*, the sample size, were

S^2	40	30	20	42	50
n	6	4	3	7	8

(1) Find a 95% confidence interval for the common variance σ^2.

(2) At level of significance $\alpha = 0.05$, conduct the following hypothesis test :

$H_0 : \sigma^2 \geq 50,$

$H_1 : \sigma^2 < 50.$

2. The result of the number preference survey is shown in the following table.

Number	1	2	3	4	5	6	7	8	9	10
Frequency	2	3	5	2	0	2	9	6	3	1

From the survey result, it is hypothesized that more than half of NTU students prefer numbers in the 6-to-10 group. Conduct a hypothesis test to show whether the above hypothesis is statistically evident at level of significance $\alpha = 0.05$.

3. The results of age guessing for 30 persons are shown in the EXCEL file 2015_Age_Guessing.csv. Thirty nine students participated in age guessing. Let X and Y represent the errors of guess for 17 females $(x_i,\ i = 1,\cdots,17)$ and 13 males $(y_i,\ i = 1,\cdots,13)$, respectively. Assuming both X and Y are normally distributed.

(1) It is suspected that, on average, female ages are underestimated. Conduct a hypothesis test at level of significance $\alpha = 0.05$ to see whether such a suspicion has a strong evidence base.

(2) Assuming that variances of X and Y are the same. Conduct the following hypothesis test

$H_0 : \mu_X - \mu_Y \geq 0$

$H_1 : \mu_X - \mu_Y < 0$

4. The data in the following table is a random sample from an exponential density $f_X(x;\ \lambda) = \lambda e^{-\lambda x}$.

62.48	10.37	31.19	9.57	85.27	162.96	16.11	133.86	47.28	34.79
70.15	75.26	25.90	9.34	47.28	4.33	35.09	48.69	83.99	158.74
60.66	56.81	64.01	89.74	7.02	64.72	100.14	16.64	147.06	28.00
116.40	11.01	29.21	12.54	43.04	18.69	50.47	17.47	56.22	25.18

(1) Conduct the following hypotheses test at level of significance $\alpha = 0.05$:

$H_0 : \lambda \leq \frac{1}{45},\quad H_1 : \lambda > \frac{1}{45}$ (considering $n\lambda\overline{X}_n$ as a pivotal quantity)

(2) Find the corresponding confidence interval of λ.

(3) Calculate and plot the power function of the test.

5. Use the same data in problem 4.

(1) Conduct the following hypotheses test at level of significance $\alpha = 0.05$:

$H_0: \lambda \leq \frac{1}{45}, \quad H_1: \lambda > \frac{1}{45}$ (considering $n\lambda Y_1$ as a pivotal quantity, where Y_1 is an order statistic.)

(2) Find the corresponding confidence interval of λ.
(3) Calculate and plot the power function of the test.
(4) Compare the powers of the tests in problems 2 and 3.

6. A random sample of size 20 ($(x_1, x_2, \cdots, x_n), \quad n = 20$) drawn from a uniform density $U[0, \theta]$ is shown in the following table.

11.90	6.94	14.90	4.36	14.00
15.21	18.87	4.75	7.67	6.57
5.15	14.83	7.79	13.88	18.55
2.49	9.90	9.68	11.10	9.92

(1) Conduct the following test at level of significance $\alpha = 0.05$:
$H_0: \theta \leq 20, \quad H_1: \theta > 20$ (considering Y_n / θ as a pivotal quantity).
(2) Calculate and plot the power function of the test.